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Please do not open this question set before you are allowed to do so.

4810-1183 Approximation and Online Algorithms with Application (Spring 2017)

Quiz Problem 1

In this problem, you will construct a problem from an optimization model, then prove that it is an NP-hard problem.

Consider the following situation.

> All-you-can-eat strawberry farms become much more popular, because many students from this course decided to go there. The farm owners are very happy about that. However, because the course instructor tells a lot about how an impolite customer can benefit from being impolite, the students are not polite. There are a lot of partly-eaten strawberries when they leave. After heavily discussed, the owner decided to charge their customers for the partly-eaten strawberry. The charge will decrease the happiness that the customer can have from those partly-eaten strawberry. Suppose that the happiness from the remaining part of the partly-eaten strawberry (if eaten) is H. The charge will decrease the happiness by $c \cdot H$, when $c \geq 0$ is an integer given as an input.
>
> In this problem, we want to eat strawberry in a way to maximize our happiness after being charged.

<u>Problem 1.1</u>: State inputs of this problem by a mathematical formulation.

<u>Problem 1.2</u>: State outputs of this problem by a mathematical formulation.

<u>Problem 1.3</u>: State constraints of this problem by a mathematical formulation.

<u>Problem 1.4</u>: State objective functions of this problem by a mathematical formulation.

<u>Problem 1.5</u>: Write a program for solving knapsack problem based on the fact that an efficient algorithm for solving the optimization model in Problems 1.1 – 1.4 is given in a library.

```
[AnswerOf1_2] YourOptimizationModel([AnswerOf1_1]);

Sets knapsack(int n, int W, int[] happiness, int[] weight){
     //write your code for knapsack here
}
```

<u>Problem 1.6</u>: Give an example for your optimization model such that there is some partly-eaten strawberry in all optimal solutions.

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Quiz Problem 2

We will continue working on your optimization model in Problem 1 in this problem. In this problem, we will develop an approximation algorithm for the model. Answering Question 1.1 – 1.4 is a prerequisite for this problem.

Question 2.1: Suppose that there is two strawberries. The first strawberry weights 1.9998 gram, and your happiness from eating the strawberry is 1.9999. The second strawberry weights 2 grams, and your happiness from eating the strawberry is 2. The charging parameter c is 20, and you cannot each more than 2 grams of strawberries.

What is the optimal solution and optimal value for your optimization model in this situation?

From the next question, *Problem A* referred to the knapsack problem for an impolite customer, where it is possible to eat a part of strawberries.

Question 2.2: In our class, we discussed about an optimal algorithm for Problem A. What is a solution obtained by applying the algorithm to the situation in Question 2.1? What is the objective value for that solution?

Question 2.3: From your answer in Question 2.2, discuss why the optimal algorithm for Problem A is ***not*** an approximation algorithm for your optimization model.

Question 2.4: For any specific output, discuss why the objective value for Problem A is no smaller than the objective value for your optimization model.

Question 2.5: By Question 2.1, discuss why the optimal value for Problem A is no smaller than the objective value for your optimization model.

From the next question, Problem B referred to the knapsack problem for a polite customer, where it is not possible to eat a part of strawberries.

Question 2.6: For a specific output of Problem B, discuss why the objective value for Problem B is equal to the objective value for your optimization model.

Question 2.7: In our class, we discussed about a 0.5-approximation algorithm for Problem B. We proved that the happiness obtained from the algorithm is at least half of the happiness obtained from Problem A's optimal solution.

Discuss why the algorithm is also a 0.5-approximation algorithm for your optimization model.

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Quiz Problem 3

In this problem, you will develop an approximation algorithm based on the deterministic rounding technique for the following problem.

> At international center for information science and technology, we have a mission to make all communications between students possible. Unfortunately, there is still a language barrier between students. Let a language ability of Student 1 and Student 2 be ℓ_1 and ℓ_2 respectively. By a research result, we found that a communication between them will happen when $\ell_1 + \ell_2$ is no smaller than a given integer $\alpha \geq 1$. We know the current language ability of all students. We increase the language ability of students so that all pairs of students can communicate together. However, increasing students' language ability is not free. If we increase a cost of a student from ℓ to ℓ', it will cost us $10000 \cdot (\ell' - \ell)$ yen. We want to minimize our cost.

Question 3.1: Suppose that we have 3 students. The language abilities of Student 1,2,3 are 2, 3, 1 respectively, and α is 5. Which students should we improve their language ability? And, how much should we improve?

Question 3.2: Construct a linear program to solve an example in Question 3.1. What is matrix $\boldsymbol{A}$, vector $\boldsymbol{b}$, and vector $\boldsymbol{c}$ for this case? What is the optimal solution obtain from the linear program?

Question 3.3: Generalize Question 3.2. Give an idea how to construct matrix $\boldsymbol{A}$, vector $\boldsymbol{b}$, and vector $\boldsymbol{c}$ from an arbitrary input of this problem. (You need not to really construct the matrix and vectors. Just give the idea how to construct them.)

From next question, we assume that the language abilities of students must be integers.

Question 3.4: Suppose that the optimal solution we have from the linear program in Question 3.3 is $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^t \in \mathbb{R}^n_{\geq 0}$. We set $x'_i \coloneqq \lfloor x_i + 0.5 \rfloor$ (x'_i is a rounding integer of x_i), and set $\boldsymbol{x}' = [x'_1, x'_2, \dots, x'_n]^t \in \mathbb{Z}^n_{\geq 0}$. Discuss why $\boldsymbol{x}'$ is a possible output for your linear program in Question 3.3.

Question 3.5: Discuss why $x'_i \leq 2 \cdot x_i$.

Question 3.6: Discuss why solving the linear program in Question 3.3 and the deterministic rounding in Question 3.4 provide us a 2-approximation algorithm for this problem.

PS: In fact, international center for information science and technology is not in charge of language nor communication.